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K-essence Explains a Lorentz Violation Experiment

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Abstract

Recently, a state of the art experiment shows evidence for Lorentz violation in the gravitational sector. To explain this experiment, we investigate a spontaneous Lorentz violation scenario with a generalized scalar field. We find that when the scalar field is nonminimally coupled to gravity, the Lorentz violation induces a deformation in the Newtonian potential along the direction of Lorentz violation.

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The pursuit of Lorentz violation has attracted increasing attention. The local Lorentz symmetry has been examined in many sectors of the standard model, including the sectors relating to photons, electrons, protons, and neutrons [1, 2, 3]. No Lorentz violation has been identified so far in these sectors. The theoretical studies of Lorentz violation can be found in [2, 4, 5].

Recently, Müller, Chiow, Herrmann, Chu, and Chung [6] performed an experiment to probe the local Lorentz symmetry in the gravitational sector. They measured the phase shift of atoms using atom interferometry. Finally, they found a more than 2σ departure of Lorentz symmetry. This result may be a signal of Lorentz symmetry violation.

In [6], the deviation from Lorentz symmetry is parametrized in the SME (standard model extension) framework [2, 5]. In SME, the Lorentz violation originates from a Lorentz violating coupling in the action

$$S_{\rm LV} = \frac{M_p^2}{2}\int d^4x\sqrt{-g}(s^{\mu\nu}R^T_{\mu\nu} + t^{\mu\nu\alpha\beta}C_{\mu\nu\alpha\beta}) \ , \tag{1}$$

where $s^{\mu\nu}$, $t^{\mu\nu\alpha\beta}$ indicate Lorentz violation in gravity, $R^T_{\mu\nu}$ is the traceless part of $R_{\mu\nu}$ and $C_{\mu\nu\alpha\beta}$ is the conformal Weyl tensor. As a result of this coupling, $s^{\mu\nu}$ and $t^{\mu\nu\alpha\beta}$ inherit the symmetries of the Ricci tensor and the Riemann curvature tensor respectively.

The Lagrangian for a nonrelativistic test particle takes the form

$$\mathcal{L}_p = \frac{1}{2}mv^2 + G\frac{Mm}{r}\left(1 + \frac{s^{jk}r^jr^k}{2r^2} + \cdots\right) \ , \tag{2}$$

where "$\cdots$" denotes terms that are irrelevant to the measurement.

If one assumes standard dispersion relation for photons, the Lorentz violating tensor $s^{\mu\nu}$ is measured to be

$$s^{XX} - s^{YY} = -(5.6 \pm 2.1)\times 10^{-9} \ . \tag{3}$$

In this article, we explain the anomaly in Eq. (3) in terms of scalar fields with a generalized kinetic term. We consider the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2R + P(X) - \frac{1}{\tilde{M}^2}\partial^\mu\phi\partial^\nu\phi R_{\mu\nu}\right) \ , \tag{4}$$

where $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, with the signature of the metric $(-,+,+,+)$, and $\tilde{M}$ is an energy scale denoting the coupling strength between ϕ and $R_{\mu\nu}$. This kind of generalized kinetic term in the action has been widely used in cosmology (see, *e.g.* [7, 8, 9, 10], and references

therein). For our purpose of breaking rotational invariance, it is important that we have a negative vacuum expectation value for X; thus $P(X)$ is a function of X only. This is guaranteed at the effective action level since we assume that there is a shift symmetry of ϕ: $\phi \to \phi + a$, and this symmetry is respected also by the coupling between the Ricci tensor and the gradient of ϕ as in Eq. (4). This action has another feature that is to retain the shift symmetry of ϕ, $\partial^\mu \phi \partial^\nu \phi R_{\mu\nu}$ is the unique nontrivial coupling between ϕ, $R_{\mu\nu}$, and $C_{\mu\nu\alpha\beta}$ when our discussion is just relevant up to the first order derivative of ϕ. The reason is that one cannot construct $t^{\mu\nu\alpha\beta}$ from the gradient of ϕ, since $\partial^\mu \phi \partial^\nu \phi$ is symmetric about indices μ, ν, while $t^{\mu\nu\alpha\beta}$ inheriting the symmetry of the Riemann curvature tensor, is antisymmetric about indices μ, ν, and α, β. Thus we only need to introduce one parameter to describe the interaction strength between ϕ and gravity in the framework of SME. On the contrary, if the Lorentz violation is induced by the vector field or tensor field [11, 12, 13, 14, 15]; then one can check that up to the first order derivative of these fields, their couplings to gravity will include both the Ricci term and the Weyl term. Therefore to parametrize these couplings, at least two parameters are necessary.

To have Lorentz violation in spacelike direction, the Hamiltonian derived from the scalar field Lagrangian $P(X)$ must have a minima at $X < 0$. Note that the $X < 0$ regime is an opposite limit compared with the ghost condensation scenario, where X has an expectation value at $X > 0$. As the expectation value of X comes from spontaneously breaking, our $X < 0$ model has equal probability to be realized compared with the ghost condensation scenario, so our model has to be considered as seriously as ghost condensation. We also find that our model has interesting cosmological implications, which are to be discussed in a forthcoming work.

We use the Lagrangian

$$P(X) = \frac{X^2}{4M^4} + \frac{1}{2}X + \frac{1}{4}M^4 \ . \tag{5}$$

To have the vacuum expectation value of X, we solve the equation of motion of ϕ with $g_{\mu\nu} = \eta_{\mu\nu}$. The equation of motion takes the form

$$\partial_\mu \left(\sqrt{-g} \left(\frac{X}{M^4} + 1 \right) g^{\mu\nu} \partial_\nu \phi \right) = 0 \ . \tag{6}$$

Note that the solution $\partial_\nu \phi = 0$ is not a stable solution, since the corresponding energy is not minimal. The stable solution of the above equation is $X = -M^4$, implying a Lorentz violation.

Without losing generality, we assume the gradient of the scalar field to be along the z direction. We have $\partial_z \phi = M^2$. Inspired by the experiment [6], the interaction strength should take the form

$$\alpha \equiv \frac{M^4}{M_p^2 \tilde{M}^2} \simeq 10^{-9} \ . \tag{7}$$

In the remainder of this article, we will show explicitly that our model can explain the proposed Lorentz violation. To do this, we will solve the perturbation equations with a point mass $m\delta(\mathbf{x})$. We find that the gravitational potential induced by m is indeed deformed in the z direction. We also find that the perturbation has positive mass squared, so the perturbation is well defined and stable.

To consider perturbations, we let $\phi(x) = M^2 z + \pi(x)$ and the perturbation of the metric

$$ds^2 = -(1+2\Phi(x))dt^2 + (1-2\Psi(x))(dx^2+dy^2) + (1-2\tilde{\Psi}(x))dz^2 \ . \tag{8}$$

The Einstein equation contains the following constraint equations

$$\begin{aligned}
&\partial_0 (M_p^2(\Psi + \tilde{\Psi}) + \frac{2M^2}{\tilde{M}^2}\chi) = 0, \\
&M_p^2(\tilde{\Psi} - \Phi) + \frac{2M^2}{\tilde{M}^2}\chi = 0 \ , \\
&M_p^2(\Psi - \Phi) + \frac{2M^2}{\tilde{M}^2}\chi + \frac{2M^4}{\tilde{M}^2}\tilde{\Psi} - \frac{2M^4}{\tilde{M}^2}(\Psi - \Phi) = 0 \ .
\end{aligned} \tag{9}$$

In the above equations, the first one can be rewritten as

$$M_p^2(\Psi + \tilde{\Psi}) + \frac{2M^2}{\tilde{M}^2}\chi = \varphi \ . \tag{10}$$

where $\varphi \equiv \varphi(\mathbf{x})$ is a function with no time dependence.

Inserting these constraint equations into the rest of Einstein equation, we derive following two independent equations of motion:

$$\nabla^2\varphi - 2\alpha\varphi_{,33} = m\delta(\mathbf{x}) \ , \tag{11}$$

$$\frac{M^2}{\tilde{M}^2}\left(-2\Psi_{,00} + \nabla^2(\Psi - \Phi) + \Box\tilde{\Psi} + (\Psi - \tilde{\Psi})_{,33}\right) + \chi + M^2\tilde{\Psi} = 0 \ , \tag{12}$$

where $\chi \equiv \pi_{,3}$, $\Box \equiv -\partial_t^2 + \partial_x^2$, and Eq. (12) is consistent with the equation of motion of π.

To solve above equations, we first express Φ, Ψ, and $\tilde{\Psi}$ in terms of φ and χ,

$$\begin{aligned}
\Phi &= \frac{1-2\alpha}{1-3\alpha}\frac{\varphi}{2M_p^2} + \frac{1-2\alpha}{1-3\alpha}\frac{M^2}{\tilde{M}^2 M_p^2}\chi \ , \\
\Psi &= \frac{1-4\alpha}{1-3\alpha}\frac{\varphi}{2M_p^2} - \frac{1-2\alpha}{1-3\alpha}\frac{M^2}{\tilde{M}^2 M_p^2}\chi \ , \\
\tilde{\Psi} &= \frac{1-2\alpha}{1-3\alpha}\frac{\varphi}{2M_p^2} - \frac{1-4\alpha}{1-3\alpha}\frac{M^2}{\tilde{M}^2 M_p^2}\chi \ .
\end{aligned} \tag{13}$$

In terms of φ and χ, Eq. (12) can be rewritten as

$$-\Box\chi - \frac{2\alpha}{3-8\alpha}\chi_{,33} + \frac{\tilde{M}^2}{2M^2(3-8\alpha)}\left(\nabla^2\varphi - 2\alpha\varphi_{,33} - 4\alpha\nabla^2\varphi\right) + m_1^2\chi - m_2^2\varphi = 0 \ , \tag{14}$$

where

$$m_1^2 \equiv \frac{(1-2\alpha)^2}{\alpha(3-8\alpha)}\tilde{M}^2 \ , \qquad m_2^2 \equiv \frac{1-2\alpha}{2(3-8\alpha)}\frac{\tilde{M}^4}{M^2} \ . \tag{15}$$

When we set $m = 0$ in Eq. (11), we have $\varphi = 0$, and the χ field has an oscillating solution with positive mass m_1, much larger than $\tilde{M}$. In other words, the perturbation of χ has positive mass squared. The perturbation is stable.

$\tilde{M}$ is the mass scale appearing in the Ricci term in Eq. (4), representing the mass scale at which this term is generated. If this term is due to quantum gravity effect, $\tilde{M}$ is close to M_p. If for some purpose we want to have a much lower scale, we will have to assume new physics (for instance a large extra dimension) from which this Ricci term arises.

We further consider gravity with source. When $m \neq 0$, the solution of Eq. (11) is

$$\varphi = -\frac{m}{\sqrt{1-2\alpha}4\pi r'} \ , \qquad r'^2 \equiv x^2 + y^2 + z'^2 \ , \qquad z' \equiv \frac{z}{\sqrt{1-2\alpha}} \ . \tag{16}$$

Inserting Eq. (16) into Eq. (14), we have

$$\begin{aligned}
\chi =& \frac{m\tilde{M}^2(1-4\alpha)}{2M^2(3-8\alpha)}\frac{e^{-m_1 r''}}{4\pi r''} - \frac{amm_2^2}{m_1\sqrt{1-2\alpha}}\int_0^1 \frac{1}{\sqrt{t}}e^{-\sqrt{t}m_1 r'''}dt \\
&- \frac{4a\alpha^2}{(3-8\alpha)\sqrt{1-2\alpha}}\frac{m\tilde{M}^2}{m_1 M^2}\partial_3^2\int_0^1 \frac{1}{\sqrt{t}}e^{-\sqrt{t}m_1 r'''}dt \ ,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
& r''^2 \equiv x^2 + y^2 + z''^2 \ , \qquad z'' \equiv \frac{z}{\sqrt{1+2\alpha/(3-3\alpha)}} \ , \\
& r'''^2 \equiv x^2 + y^2 + z'''^2 \ , \qquad z''' \equiv \frac{z''}{\sqrt{t+(1-t)a^2}} \ , \qquad a \equiv \sqrt{\frac{(1-2\alpha)(3-3\alpha)}{3-\alpha}} \ .
\end{aligned} \tag{18}$$

To the first order in α, Eq. (17) takes the form

$$\frac{M^2}{\tilde{M}^2 M_p^2}\chi = \frac{1}{3}\frac{Gme^{-m_1 r''}}{r''} - \frac{\alpha G m}{r}\left(1 - e^{-m_1 r}\right) \ . \tag{19}$$

Note that the contribution from χ to the gravitation potential Φ is either suppressed by the Yukawa factor $e^{-m_1 r''}$ or by the small number α. So at a long distance, the only contribution from χ to Φ is a shift in the Newtonian constant G. Insert Eqs. (16) and (19) into Eq. (13), and we have

$$\Phi = -(1+3\alpha)\frac{Gm}{r}\left(1 - \frac{z^2}{r^2}\alpha\right) \ , \tag{20}$$

where $G \equiv 1/(8\pi M_p^2)$, and the factor $1+3\alpha$ can be absorbed into a redefinition of G, so it is not measurable. Meanwhile the term $1-\frac{z^2}{r^2}\alpha$ gives an explicit Lorentz violation. Comparing with (2), we have

$$s^{33} = -2\alpha \ . \tag{21}$$

To compare with experiments, we can identify the third direction (denoted by z or 3 in the article) with the X direction in [6]. Then a value $\alpha = 2.8 \times 10^{-9}$ gives an explanation to the measurement [6]. Alternatively, we can also identify the z direction with the Y direction in [6], and let $\alpha = -2.8 \times 10^{-9}$ to explain the experiment. In this case, $\tilde{M}^2 < 0$, while the perturbation is still stable.

Theoretically, we find that physics at string scale may be responsible for the small value of α. The reason is below. As mentioned before, it is reasonable to assume that the scalar-gravity coupling term in Eq. (4) originates from the quantum gravity effects, so $\tilde{M} \simeq M_p$. Then from the expression of α Eq. (7), we read

$$M = |\alpha|^{1/4} M_p \simeq 1.7 \times 10^{16}\mathrm{GeV}, \tag{22}$$

where $M_p \simeq 2.4 \times 10^{18}\mathrm{GeV}$ has been used (M_p is the reduced Planck mass defined as $M_p \equiv 1/\sqrt{8\pi G}$). The energy scale of M can naturally arise from string theory by requiring the scale of extra dimension approach Planck scale. At this stage, we cannot guarantee the Lorentz violation is due to stringy effects, but there is the possibility that Lorentz violation is induced by some stringy physics effectively described by the generalized scalar field.

Finally, we consider some signatures of our model at small length scales. At small length scales, the first term in the right hand side of Eq. (19) becomes important. Combining this term and the contribution from φ, we will obtain a gravitation potential with a running

Newtonian constant,

$$\Phi = -\frac{G(r)m}{r}, \tag{23}$$

and

$$G(r) = (1 - \frac{1}{3}e^{-m_1 r})G, \tag{24}$$

where we have neglected the terms proportional to α. As we remarked before, $\tilde{M}$ is the energy scale at which the Ricci term is generated, so naturally it is not small, while m_1 is a factor $1/\alpha$ larger than $\tilde{M}$, an even larger mass scale, so it is not conceivable to measure the running of the Newton constant.

To conclude, we have considered spontaneous Lorentz violation from a generalized scalar field. We show that when X has a nonzero vacuum expectation value, the Lorentz symmetry is spontaneously broken. When coupling to gravity, this Lorentz violation affects the Newtonian potential. This modification of gravity can explain the current experiment [6], and can be tested in future experiments. As stated in [6], future experiments may reach the accuracy of 10^{-14}. It is very interesting to see whether the Lorentz violation is confirmed in the future.

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